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RELIABILITY PAPER

Six Sigma quality evaluation of life test data based on Weibull distribution

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Abstract

Purpose – While Six Sigma metrics have been studied by researchers in detail for normal distribution-based data, in this paper, we have attempted to study the Six Sigma metrics for two-parameter Weibull distribution that is useful in many life test data analyses.

Design/methodology/approach – In the theory of Six Sigma, most of the processes are assumed normal and Six Sigma metrics are determined for such a process of interest. In reliability studies non-normal distributions are more appropriate for life tests. In this paper, a theoretical procedure is developed for determining Six Sigma metrics when the underlying process follows two-parameter Weibull distribution. Numerical evaluations are also considered to study the proposed method.

Findings – In this paper, by matching the probabilities under different normal process-based sigma quality levels (SQLs), we first determined the Six Sigma specification limits (Lower and Upper Six Sigma Limits- LSSL and USSL) for the two-parameter Weibull distribution by setting different values for the shape parameter and the scaling parameter. Then, the lower SQL (LSQL) and upper SQL (USQL) values are obtained for the Weibull distribution with centered and shifted cases. We presented numerical results for Six Sigma metrics of Weibull distribution with different parameter settings. We also simulated a set of 1,000 values from this Weibull distribution for both centered and shifted cases to evaluate the Six Sigma performance metrics. It is found that the SQLs under two-parameter Weibull distribution are slightly lesser than those when the process is assumed normal.

Originality/value – The theoretical approach proposed for determining Six Sigma metrics for Weibull distribution is new to the Six Sigma Quality practitioners who commonly deal with normal process or normal approximation to non-normal processes. The procedure developed here is, in fact, used to first determine LSSL and USSL followed by which LSQL and USQL are obtained. This in turn has helped to compute the Six Sigma metrics such as defects per million opportunities (DPMOs) and the parts that are extremely good per million opportunities (EGPMOs) under two-parameter Weibull distribution for lower-the-better (LTB) and higher-the-better (HTB) quality characteristics. We believe that this approach is quite new to the practitioners, and it is not only useful to the practitioners but will also serve to motivate the researchers to do more work in this field of research.

Keywords DPMO, EGPMO, Non-normal process, Six sigma metrics, Weibull process

Paper type Research paper

1. Introduction

In this world of conspicuous consumers, the manufacturers aim to become increasingly agile so as to compete in the global market by designing their product as expected by the customers. Further, it is the responsibility of the firm to make sure that their customers do not switch over to buy similar products from other competing manufacturers. Apart from building long-term customer relationships, attracting new customers by providing the insights of quality in the product is an effective strategy for the manufacturer to increase the business. In fact, it is worth mentioning that quality monitoring has a direct impact on the

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bottom-line of any business organization. In this process, introduced by Motorola in 1980s, Six Sigma quality initiative has evolved with a great zeal to ensure quality and customer satisfaction. This can be evidenced by the availability of a number of research activities in the field of Six Sigma quality and its applications. For example, readers are referred to the works of McFadden (1993), Harry (1998), Lucas (2002), Antony *et al.* (2007), Aboelmaged (2010), Tjahjono (2010), Setijono (2008, 2010), Shruti and Ravi Kant (2017), Ravichandran (2006, 2016, 2017, 2019), Schroeder *et al.* (2008).

It is interesting to note that in statistical process control (SPC) and in Six Sigma applications, the quality characteristics of products and processes are often analyzed taking into consideration that the related data are normal. Accordingly, out-of-control situations or defect rates are computed (McFadden, 1993; Chou *et al.*, 1998; Lucas, 2002; Ravichandran, 2006). However, it is experienced and argued by many researchers that though normality assumption is reasonable to processes that deal with large sets of data, such assumption may end up in undesirable results, if the data are not following normal distribution (e.g. Antony, 2004; Chou *et al.*, 1998; Setijono, 2008, 2010; Aldowaisan *et al.*, 2015).

In this regard, it is often suggested to transform the non-normal data into normal data with the available methods and then to deal with the data as if it is from the normal process (e.g. Chou *et al.*, 1998). Setijono (2008, 2010) studied the use of some normal approximation methods to non-normal data for evaluating the defect rates in terms of defects per million opportunities (DPMO), a key performance measure for Six Sigma quality process. Yap and Sim (2011) compared the types of various normality tests. It may be noted that there are occasions where one may experience two-sided inference with equal tail probabilities which is not appropriate even if the data are normally distributed. Ceyhun (2016) studied such issues elaborately with the application of normal and t-distributions to hypothesis testing considering unequal Type-I error probabilities.

McFadden (1993) presented the Six Sigma metrics elaborately when the underlying distribution is normal. The defect rates such as parts per million (PPM) and DPMO along with the process capability of both centered and shifted process cases are very well explained by McFadden (1993). Hassan *et al.* (2018) studied the features of Six Sigma using a case study in oil and gas where process is assumed as gamma-distributed. Aldowaisan *et al.* (2015) considered a case example of oil companies' supplier bills and studied the aspects of Six Sigma performance taking into account the process of interest is non-normal. They considered the case with exponential process and demonstrated the difference in using the exponential data as it is instead of using normal approximation. Aldowaisan *et al.* (2015) presented the failure rates with different sigma levels for Weibull distribution as well. Recently Ravichandran (2019) studied the Six Sigma metrics based on lognormal distribution.

Setijono (2008, 2010) considered the need for using normal approximation methods to non-normal data in case of dealing with customer satisfaction survey data for evaluating the defect rates in terms of DPMO. Originally, Setijono (2008) introduced the concepts of dissatisfaction per million opportunities abbreviated as (DisPMO) and delight per million opportunities abbreviated as (DePMO). The DisPMO and DePMO represent the cases (customer reactions as dissatisfied or delighted) below the Lower Six Sigma Limit (LSSL) and above the Upper Six Sigma Limit (USSL) respectively. Ravichandran (2016) studied the problem of estimating DPMO and the parts that are extremely good per million opportunities (EGPMO) for quality evaluation from the perspective of higher-the-better and lower-the-better quality characteristics. In his work, Ravichandran (2006) assumed that the process follows normal distribution and based on that the Sigma Quality Limits (SQLs) are obtained.

In the present work, we develop a theoretical procedure for determining Six Sigma metrics (DPMO, EGPMO and SQLs) considering the process distribution as it is (i.e. the two-parameter Weibull distribution in this case) rather than considering normal approximation to it. It may be noted that both Aldowaisan *et al.* (2015) and Hassan *et al.* (2018) considered zero as the target and computed sigma deviations and shifts accordingly after transforming it into normal distribution. In our approach, we use the theoretical procedure to first determine LSSL and USSL followed by which the lower and upper SQLs (LSQL and USQL) are obtained so that DPMOs and/or EGPMOs can be computed for lower-the-better (LTB) and higher-the-better (HTB) quality characteristics of the Weibull distribution. Clearly, this proposed procedure keeps the actual target rather than keeping zero as the target as done by Aldowaisan *et al.* (2015) and Hassan *et al.* (2018). The proposed approach is, in fact, quite new and useful to the Six Sigma quality practitioners who commonly deal with normal processes or normal approximations to non-normal processes. We believe that the proposed approach will also serve to motivate the researchers to do more work in this field of research.

The remainder of the paper is organized as follows: Section 2 describes the Six Sigma metrics for Weibull distribution in which the Six Sigma metrics from the general perspective of normal distribution assumption is discussed first followed by this a procedure for determining the proposed Six Sigma metrics for Weibull distribution is given in detail. The Sigma limits (LSSL and USSL) for both centered and shifted Weibull processes are also studied in detail. This section includes the procedure for obtaining SQLs also. In Section 3, the proposed computational procedures for obtaining DPMO/EGPMO values with higher-the-better (HTB) and lower-the-better (LTB) quality characteristics when the underlying process is two-parameter Weibull are presented. In Section 4, the proposed Six Sigma metrics of Weibull distribution are numerically evaluated followed by which the simulated data are used to obtain DPMO/EGPMO for illustration. The paper ends in Section 5 with discussions based on the results obtained.

2. Six sigma metrics for Weibull distribution

2.1 Six sigma metrics from general perspective

Six Sigma extensively uses the normal distribution to calculate the defect rates (DPMOs) in the process and works on it to minimize the defect percentage, thereby enhancing the quality of the product. But in real-time processes, the data need not be distributed normally always. For example, in life data analysis, the analyst attempts to predict the life of the product in the population data by estimating the Critical to Quality (CTQ) characteristic(s) by knowing what is important to the customer. Initially, to characterize the capability of a process, the following measures of capability indices (C_p and C_{pk}) are used:

$$C_p = \frac{\text{USL} - \text{LSL}}{6\sigma} \quad \text{and} \quad C_{pk} = \min\left\{\frac{\text{USL} - \mu}{3\sigma}, \frac{\mu - \text{LSL}}{3\sigma}\right\} \tag{1}$$

where C_p is computed for the process centered at the target μ at design stage and C_{pk} for the target with shifted mean $\mu \pm K\sigma$, K is constant greater than or equal zero and σ is the standard deviation of the process. In Eqn (1), LSL represents the lower specification limit and USL represents the upper specification limit. McFadden (1993) presented the Six Sigma metrics elaborately when the underlying distribution is normal. The study by McFadden (1993) includes the cases of process with and without shifts. Table 1 presents the process capability

	Centered process		Shifted process	
SQL of normal process	c_p	DPMO	c_{pk}	DPMO
3	1	2,700	0.5	66803
4	1.33	63	0.833	6,200
5	1.67	0.57	1.167	233
6	2	0.002	1.5	3.4

Table 1. Process capability values and the corresponding DPMOs under normality assumption

values and the corresponding DPMOs for different SQLs when the process is either centered or shifted. It is important to note that for a Six Sigma process with shift in the mean up to $\pm 1.5\sigma$ will still yield only 3.4 DPMO, and that is why MOTOROLA and other firms allowed the process to the acceptable drift up to $\pm 1.5\sigma$.

For non-normal distributions, either skewed or heavy tailed distributions, these process capability measures and Six Sigma metrics such as DPMO and EGPMO are likely to lead unexpected erroneous results. Also, for non-normal data, if normal distribution metrics are used, it may lead to undesirable results and hence care should be taken when dealing with non-normal data (Antony, 2004). It may also be noted that even if the data follow normal distribution, the tail probabilities need not be equal (Ceyhun, 2016).

Wooluru *et al.* (2016) discussed the use of various methods in determining process capability measures for non-normal distributions. Aldowaisan *et al.* (2015) considered a case example of oil companies' supplier bills and studied the aspects of Six Sigma performance taking into account the process of interest is non-normal. They considered the case with exponential process and demonstrated the difference in using the exponential data as it is, instead of using normal approximation. Aldowaisan *et al.* (2015) presented the failure rates with different SQLs for Weibull distribution as well taking into consideration the sigma deviations and process shifts are computed keeping zero as the target.

As discussed in the introduction section, Setijono (2008, 2010) considered LSSL and USSL at $\mu \pm 6\sigma$ where μ is the average customer response and σ is the standard deviation of the responses such that the probability that a response will fall outside the limits is 2.00×10^{-9} for a centered process and 3.4×10^{-6} for a shifted process with shift in the mean up to $\pm 1.5\sigma$. For more details readers are referred to Lucas (2002), Antony (2004) and Ravichandran (2016). In the work of Setijono (2008, 2010), while DisPMO corresponds to the area $< \mu - 6\sigma$, the DePMO are considered from the area $> \mu + 6\sigma$. This means that the DisPMO and DePMO represent the customer responses below LSSL and above USSL respectively.

According to Ravichandran (2016), under normality assumption, for HTB case, while DPMO corresponds to the area $< \mu - 6\sigma$, the EGPMO corresponds to the area $> \mu + 6\sigma$ and these areas are $> \mu + 6\sigma$ and $< \mu - 6\sigma$ for DPMO and EGPMO respectively for LTB case. Therefore, the probability that the quality characteristic, say X, is considered in such a way that $P(\mu - 6\sigma \leq X \leq \mu + 6\sigma) = 1 - (2.00\times10^{-9})$ for a centered process and is $P(\bar{X} - 4.5\sigma/\sqrt{n} \leq X \leq \bar{X} + 4.5\sigma/\sqrt{n}) = 1 - (6.8\times10^{-6})$ for a shifted process with shift in the mean up to $\pm 1.5\sigma$ where $\bar{X}$ is the sample mean based on a sample of size n.

According to Setijono (2008, 2010) and Ravichandran (2016), there are situations where only left-tail or right-tail DPMOs are treated as favorable. Therefore, simultaneous evaluation of DPMO (or DisPMO) and EGPMO (or DePMO) is necessitated to get details about the dissatisfied customers (or defective product units) and delighted customers (or extremely good product units). In general, since customer data are non-normal, Setijono (2008, 2010) computed DisPMO and DePMO using various normal approximation methods. In these lines, Ravichandran (2016) proposed an approach for estimating EGPMO and DPMO for HTB and LTB cases under normality assumption. It is suggested that in case of non-normal process, there is a scope for determining these metrics (DPMO and EGPMO) taking into consideration the true distribution itself instead of normal approximations and this is the primary motivation for the authors to take up this research.

2.2 Six sigma metrics for Weibull distribution

In this paper, the two-parameter Weibull distribution with parameters k (shape parameter) and β (scale parameter) is considered which is the most commonly used non-normal distribution in the field of reliability engineering for studying life test data. In fact, in many practical examples like survival analysis, weather forecasting, wind speed distributions in wind power industry, modeling the size of reinsurance claims in general insurance schemes, the data are observed to follow Weibull distribution. Weibull distribution is, in fact, the best fitted model based on maximum likelihood criterion for such data. A continuous random variable X is said to follow Weibull distribution with parameters k and β if its probability density function is given by,

$$f(x) = \begin{cases} \frac{k}{\beta^k} e^{-\left(x/\beta\right)^k} x^{k-1}, 0 \le x < \infty; \quad k, \beta > 0 \\ 0, \text{ otherwise} \end{cases} \tag{2}$$

The mean and variance of Weibull distribution can be expressed as

$$\text{Mean } \mu_w = \beta \Gamma\left(1 + \frac{1}{k}\right) \tag{3}$$

$$\text{Variance } \sigma_w^2 = \beta^2 \left[\Gamma\left(1 + \frac{2}{k}\right) - \left\{\Gamma\left(1 + \frac{1}{k}\right)\right\}^2\right] \tag{4}$$

2.2.1 SQL, DPMO and EGPMO for centered Weibull process. In this paper, it is attempted to evaluate the Six Sigma metrics (DPMO, EGPMO and SQLs) for two-parameter Weibull distributed data. Given the quality characteristic X, the two tail-probabilities α_1 and α_2 under Weibull distribution can be given as

$$\alpha_1 = P\left(X < W_{\alpha_1}\right) \text{ and } \alpha_2 = P\left(X > W_{\alpha_2}\right) \text{ such that } \alpha_1 + \alpha_2 = \alpha = 2.00 \times 10^{-9} \tag{5}$$

where W_{α_1} and W_{α_2} are respectively the values corresponding to α_1 and α_2 percentiles in the cumulative distribution of X. Hence, for a Six Sigma Weibull process we have

$$\alpha_1 + \alpha_2 = \alpha = 2.00 \times 10^{-9} \tag{6}$$

It may be noted that α_1 and α_2 may or may not be equal. If $\alpha_1 = \alpha_2$, then

$$\alpha_1 = \alpha_2 = \frac{\alpha}{2} = 1.00 \times 10^{-9} \tag{7}$$

Accordingly, for the centered Weibull process, the DPMO or EGPMO can be obtained as

$$\begin{aligned} \text{DPMO or EGPMO(left tail)} &= \alpha_1 \times 10^6 \\ \text{DPMO or EGPMO(right tail)} &= \alpha_2 \times 10^6 \end{aligned} \tag{7a}$$

Therefore, with the process centered at the target mean μ_w defined in Eqn (3) the Six Sigma Limits (LSSL and USSL) for the Weibull quality characteristic X can be given as

$$P\left\{W_{\alpha_1}(C) \le X \le W_{\alpha_2}(C)\right\} = P(\text{LSSL}_c \le X \le \text{USSL}_c) = 1 - \left(2.00 \times 10^{-9}\right) \tag{8}$$

where LSSL_c and USSL_c represent LSSL and USSL respectively of the centered process, and

$$\begin{aligned} W_{\alpha_1}(C) &= \text{LSSL}_c = \mu_w - K_{c1}\sigma_w \\ W_{\alpha_2}(C) &= \text{USSL}_c = \mu_w + K_{c2}\sigma_w \end{aligned} \tag{9}$$

$$\text{Where } K_{c1} = \frac{\mu_w - W_{\alpha_1}(C)}{\sigma_w} \text{ and } K_{c2} = \frac{W_{\alpha_2}(C) - \mu_w}{\sigma_w} \tag{10}$$

$$\Rightarrow K_{c1} + K_{c2} = \frac{W_{\alpha_2}(C) - W_{\alpha_1}(C)}{\sigma_w} \tag{11}$$

here, K_{c1} is the lower SQL (LSQL) and K_{c2} is the upper SQL (USQL) for the centered process. This implies that when a centered Six Sigma normal process has SQL = 6, then the corresponding SQL of the centered Weibull process will be $(K_{c1} + K_{c2})/2$ on the average. For better understanding, a flow diagram is given in Figure 1 to show how SQL, DPMO and EGPMO for a centered two-parameter Weibull process are computed.

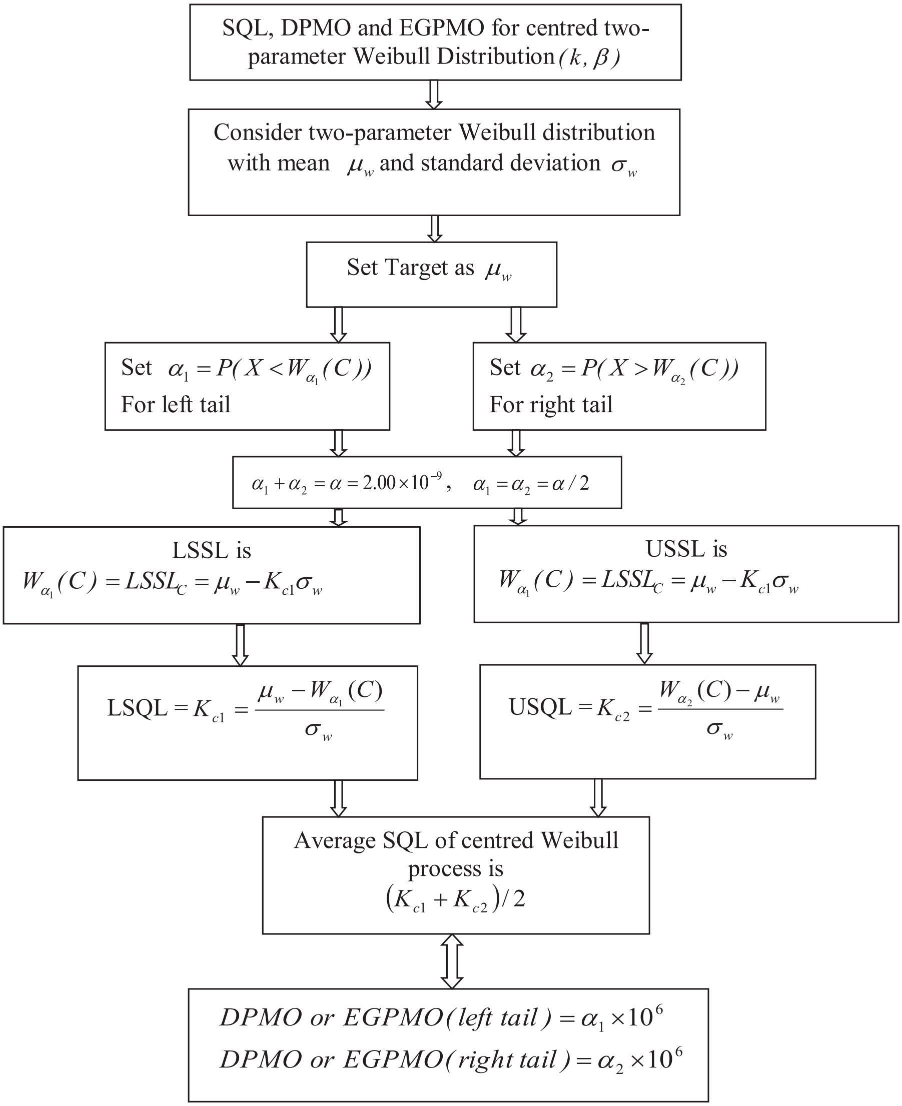

Figure 1.
Procedure for obtaining SQL, DPMO and EGPMO for a centered Weibull process

2.2.1 SQL, DPMO and EGpMO for shifted Weibull process. It may be noted that though it is not preferable, in the long run there is always a scope for the process to experience a shift in the process mean up to ±1.5 times of standard deviation (Lucas, 2002; Antony, 2004; Ravichandran, 2006). Such a shift can happen either toward right tail or toward left tail of the process distribution. In their study on Six Sigma performance of non-normal processes, Aldowaisan *et al.* (2015) who considered exponential-based process and Hassan *et al.* (2018) who considered gamma-based process, argued that such a shift toward left is inconsequential as the shifted mean may be below zero which is not permissible for such distributions. Therefore, they studied the processes with shifts toward right tail only taking into consideration zero as the target.

We consider a shift of $1.5\sigma_w$ to the mean of the centered Weibull process toward right tail and also toward left tail (if the shifted mean is greater than zero). This aspect will help to accommodate HTB and LTB-type quality characteristics based on which DPMO and/EGPMO can be obtained. The shifted mean for Weibull process is now given as

$$\bar{X}_w = \mu_w \pm 1.5\sigma_w \tag{12}$$

now, with shifted mean $\bar{X}_w$ and standard deviation σ_w, the parameters, say k' (shape) and β' (scale) of the shifted Weibull distribution are computed (refer to Eqn (24)) using which the left-tail probabilities α_1' and α_2' are obtained as:

$$\alpha_1' = P\left\{X < W_{\alpha_1'}(C) \big/ k', \beta'\right\} \text{ and } \alpha_2' = P\left\{X > W_{\alpha_2'}(C) \big/ k', \beta'\right\} \tag{13}$$

Therefore, for the shifted Weibull process, the DPMO or EGPMO can be obtained as

$$\begin{aligned} \text{DPMO or EGPMO(left tail)} &= \alpha_1' \times 10^6 \\ \text{DPMO or EGPMO(right tail)} &= \alpha_2' \times 10^6 \end{aligned} \tag{14}$$

now, the SQLs of the shifted Weibull process can be computed as follows:

$$K_{s1} = \frac{\bar{X}_w - W_{\alpha_1'}(C)}{\sigma_w} \text{ and } K_{s2} = \frac{W_{\alpha_2'}(C) - \bar{X}_w}{\sigma_w} \tag{15}$$

where K_{s1} is the LSQL and K_{s2} is the USQL for the shifted process. Therefore, while SQL = 4.5 for the shifted normal process, the SQL of the shifted Weibull process will be $(K_{s1} + K_{s2})/2$ on the average. For better understanding, a flow diagram is given in Figure 2 to show how the SQL for a shifted Weibull process is computed.

3. Computation of SQL, DPMO and EGPMO values

Aldowaisan *et al.* (2015) and Hassan *et al.* (2018) considered a case example of Oil Company with a time limit of 30 days for processing of bills. They studied the Six Sigma metrics for normal, Gamma and Exponential-distributed processes in which the DPMO are computed and compared. In these studies, the two-sided specification limits under normal process are converted into one-sided (right side) specification limits under Gamma and Exponential processes for the evaluation of DPMO. Setijono (2010) and Ravichandran (2016) argued about the situations in life test studies where both the tail probabilities are important to determine DPMO and EGPMO, particularly when the quality characteristic is of HTB or LTB type. Setijono (2010) classified the customer response as DisPMO to represent dissatisfied respondents (customers) when average response is falling below the LSSL and as DePMO to represent delighted respondents (customers). It is, in fact, a case of HTB category. However, Ravichandran (2016) demonstrated that there are situations such as testing of bursting

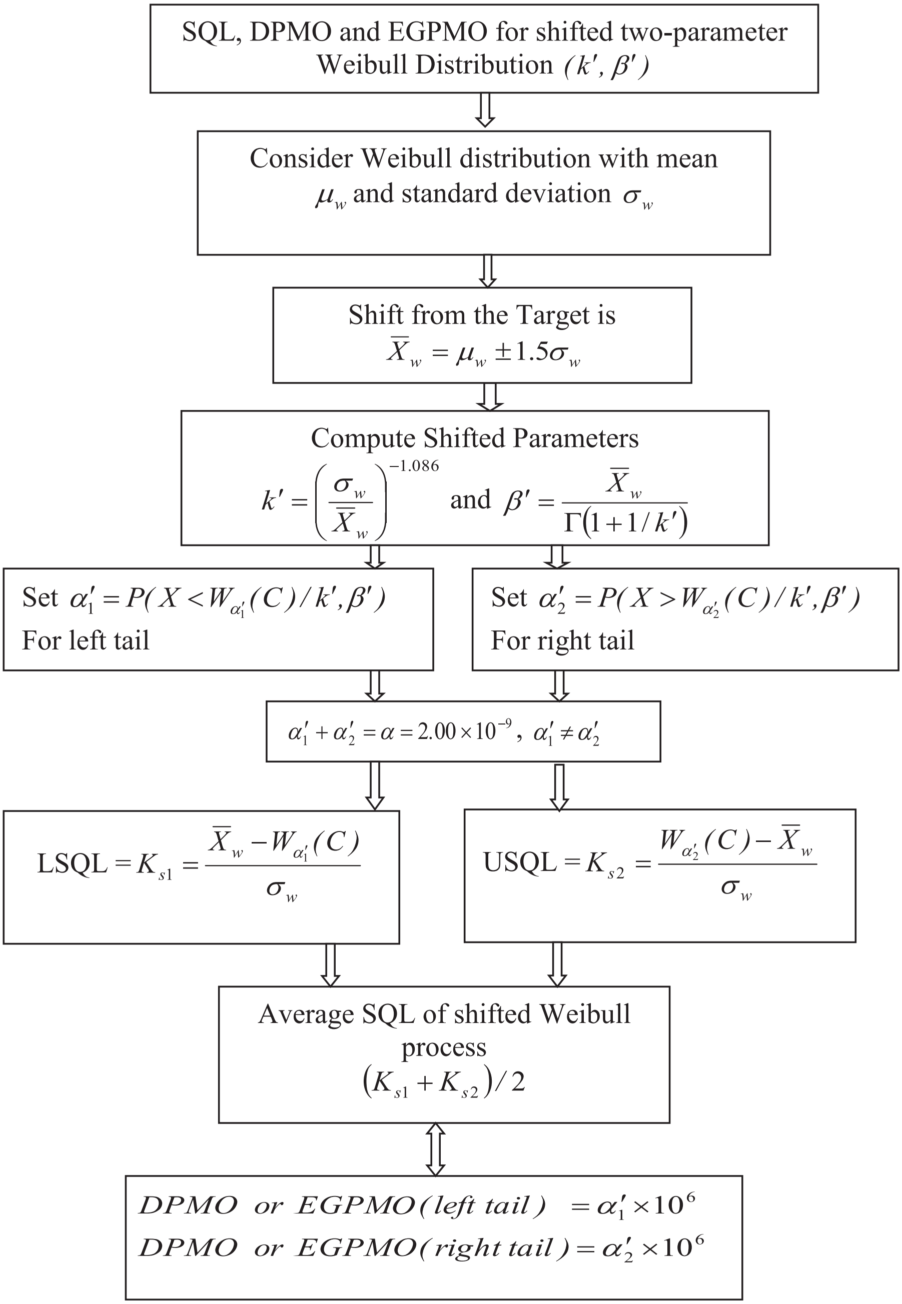

Figure 2.
Procedure for obtaining SQL, DPMO and EGPMO for a shifted Weibull process

strength where HTB is applicable and testing of weights of components for aircraft requirements where LTB is applicable. Therefore, in this paper unlike Aldowaisan *et al.* (2015) and Hassan *et al.* (2018), both LTB and HTB cases are considered from the general perspective rather than with a specific oil company example.

Under a centered process (no-shift case), we express the defective units x_{d_i} and extremely good units x_{eg_i} for HTB Eqn (16) and LTB Eqn (17) cases, as

$$\begin{aligned} x_{d_i} &= \begin{cases} 1, x_i < W_{\alpha_1}(C)|k,\beta \\ 0, x_i > W_{\alpha_1}(C)|k,\beta \end{cases} \quad i = 1,2,3,\cdots,n \\ x_{eg_i} &= \begin{cases} 1, x_i > W_{\alpha_2}(C)|k,\beta \\ 0, x_i < W_{\alpha_2}(C)|k,\beta \end{cases} \quad i = 1,2,3,\cdots,n \end{aligned} \tag{16}$$

$$\begin{aligned} x_{d_i} &= \begin{cases} 1, x_i > W_{\alpha_2}(C)|k,\beta \\ 0, x_i < W_{\alpha_2}(C)|k,\beta \end{cases} \quad i = 1,2,3,\cdots,n \\ x_{eg_i} &= \begin{cases} 1, x_i < W_{\alpha_1}(C)|k,\beta \\ 0, x_i > W_{\alpha_1}(C)|k,\beta \end{cases} \quad i = 1,2,3,\cdots,n \end{aligned} \tag{17}$$

Similarly, under shifted process (with-shift case), the defective units x_{d_i} and extremely good units x_{eg_i} for HTB Eqn (18) and LTB Eqn (19) cases are given as, as

$$\begin{aligned} x_{d_i} &= \begin{cases} 1, x_i < W_{\alpha_1'}(C)|k', \beta' \\ \\ 0, x_i > W_{\alpha_1'}(C)|k', \beta' \end{cases} \quad i = 1,2,3,\cdots,n \\ x_{eg_i} &= \begin{cases} 1, x_i > W_{\alpha_2'}(C)|k', \beta' \\ \\ 0, x_i < W_{\alpha_2'}(C)|k', \beta' \end{cases} \quad i = 1,2,3,\cdots,n \end{aligned} \tag{18}$$

$$\begin{aligned} x_{d_i} &= \begin{cases} 1, x_i > W_{\alpha_2'}(C)|k', \beta' \\ \\ 0, x_i < W_{\alpha_2'}(C)|k', \beta' \end{cases} \quad i = 1,2,3,\cdots,n \\ x_{eg_i} &= \begin{cases} 1, x_i < W_{\alpha_1'}(C)|k', \beta' \\ \\ 0, x_i > W_{\alpha_1'}(C)|k', \beta' \end{cases} \quad i = 1,2,3,\cdots,n \end{aligned} \tag{19}$$

Now, DPMO which is defined as the ratio of the number of defective units in one million opportunities and EGPMO which is defined as the ratio of the number of extremely good units in one million opportunities can be computed as follows:

$$\begin{aligned} \text{DPMO} &= \frac{n_1}{n} \times 10^6 \\ \text{EGPMO} &= \frac{n_2}{n} \times 10^6 \end{aligned} \tag{20}$$

$$n_1 = \sum_{i=1}^{n} x_{d_i} \text{ and } n_2 = \sum_{i=1}^{n} x_{eg_i} \tag{21}$$

If we define

$$p_1 = \text{DPMO} \times 10^{-6} \text{ and } p_2 = \text{EGPMO} \times 10^{-6} \tag{22}$$

then corresponding to DPMO and EGPMO, the actual Six Sigma limits, say $W_{\alpha_1'}(S)$ and $W_{\alpha_2'}(S)$ can be obtained using the inverse Weibull function IDF.WEIBULL (p, k, β) available in SPSS where k and β (or k' and β' for shifted case) are the parameters of the distribution and p may be either p_1 or p_2 given in (22). Finally the actual SQLs can be computed similar to that of Eqn (15) as

$$K_{sql1} = \frac{\bar{X}_w - W_{\alpha_1'}(S)}{\sigma_w} \text{ and } K_{sql2} = \frac{W_{\alpha_2'}(S) - \bar{X}_w}{\sigma_w} \quad (23)$$

4. Numerical results

In this section the Six Sigma metrics (SQL, DPMO and EGPMO) for two-parameter Weibull distribution as presented in Section 3 are numerically studied. The DPMO and EGPMO are evaluated from the data simulated using Weibull parameters as well. It may be noted that the failure rates are altered only by the shape parameter (k) and hence the changes made to the scaling parameter (β) has no effect on the failure rates (Aldowaisan *et al.*, 2015). Accordingly, we fix β at an arbitrary value of 2 in all our calculations. Six sets of data with the parameters considered, and the corresponding mean μ_w, variance σ_w^2 and standard deviation σ_w of the corresponding distributions are given in Table 2.

$$\text{Data set 1: } k = 0.5, \beta = 2; \text{Data set 2: } k = 1, \beta = 2$$
$$\text{Data set 3: } k = 1.5, \beta = 2; \text{Data set 4: } k = 2, \beta = 2$$
$$\text{Data set 5: } k = 2.5, \beta = 2; \text{Data set 6: } k = 3, \beta = 2$$

Table 3 gives various sigma levels, DPMOs and the corresponding sums of the tail probabilities based on normality assumption. In order to match the SSQ metrics of normal process with Weibull process, we initially compute centered SQLs, i.e. $W_{\alpha_1}(C)$ and $W_{\alpha_2}(C)$ corresponding to α_1 and α_2 using the relationship given in (5). For demonstration purpose, we consider different SQLs (3, 4, 5 and 6) of normal process (centered) and their corresponding combinations of α_1 and α_2 as given in Table 4, for computing $W_{\alpha_1}(C)$ and $W_{\alpha_2}(C)$. Now, the SQLs for Weibull process are obtained using (9) and given in Table 4.

Table 2. Mean and variance of Weibull distribution based on parameter values

Dataset	Parameters		Mean and variance based on parameter values		
	k	β	μ_w	σ_w^2	σ_w
1	0.5	2	4.0000	80.0000	8.9443
2	1.0	2	2.0000	4.0000	2.0000
3	1.5	2	1.8055	1.5028	1.2259
4	2.0	2	1.7728	0.8571	0.9258
5	2.5	2	1.7745	0.5766	0.7593
6	3.0	2	1.7684	0.4074	0.6382

Table 3. SQLs, DPMOs and sum of the tail probabilities based on normality assumption

SQL of normal process	DPMO	Sum of tail probabilities $\alpha = \alpha_1 + \alpha_2$
6.0	0.002	2.00×10^{-9}
5.5	0.038	3.80×10^{-8}
5.0	0.57	5.70×10^{-7}
4.5	6.8	6.80×10^{-6}
4.0	63	6.30×10^{-5}
3.5	465	4.65×10^{-4}
3.0	2,700	2.70×10^{-3}

SQL of normal process	α_1	α_2	LSSL $W_{\alpha_1}(C)$	USSL $W_{\alpha_2}(C)$	LSQL K_{c1}	USQL K_{c2}	Average SQL of Weibull process
6	0	2.00×10^{-9}	0	5.43156	2.770484	5.738932	4.2547
	0.50×10^{-9}	1.50×10^{-9}	0.00159	5.45744	2.767993	5.779477	4.2737
	1.00×10^{-9}	1.00×10^{-9}	0.002	5.4935	2.767351	5.835971	4.3017
	1.50×10^{-9}	0.50×10^{-9}	0.00229	5.55408	2.766896	5.930879	4.3489
	2.00×10^{-9}	0	0.00252	5.68986	2.766536	6.143600	4.4551
5	0	5.70×10^{-7}	0	4.86259	2.770484	4.847548	3.8090
	1.43×10^{-7}	4.30×10^{-7}	0.01047	4.89481	2.754081	4.898026	3.8261
	2.87×10^{-7}	2.87×10^{-7}	0.01319	4.93952	2.749820	4.968071	3.8589
	4.30×10^{-7}	1.43×10^{-7}	0.0151	5.01418	2.746828	5.085038	3.9159
	5.70×10^{-7}	0	0.01661	5.05205	2.744462	5.144368	3.9444
4	0	6.30×10^{-5}	0	4.26049	2.770484	3.904261	3.3374
	1.58×10^{-5}	4.75×10^{-5}	0.05022	4.30234	2.691806	3.969826	3.3308
	3.17×10^{-5}	3.17×10^{-5}	0.06328	4.35998	2.671346	4.060128	3.3657
	4.75×10^{-5}	1.58×10^{-5}	0.07244	4.45513	2.656995	4.209196	3.4331
	6.30×10^{-5}	0	0.07973	4.51605	2.645574	4.304637	3.4751
3	0	2.70×10^{-4}	0	3.61691	2.770484	2.895989	2.8332
	6.75×10^{-3}	2.02×10^{-3}	0.17546	3.67463	2.495598	2.986417	2.7410
	1.35×10^{-3}	1.35×10^{-3}	0.22109	3.75302	2.424111	3.109228	2.7667
	2.02×10^{-3}	6.75×10^{-3}	0.25311	3.87991	2.373946	3.308021	2.8410
	2.70×10^{-9}	0	0.27861	4.01854	2.333997	3.525208	2.9296

Table 4. Computation of SQLs for centered Weibull distribution with $k = 3$ and $\beta = 2$ for different combinations of α_1 and α_2

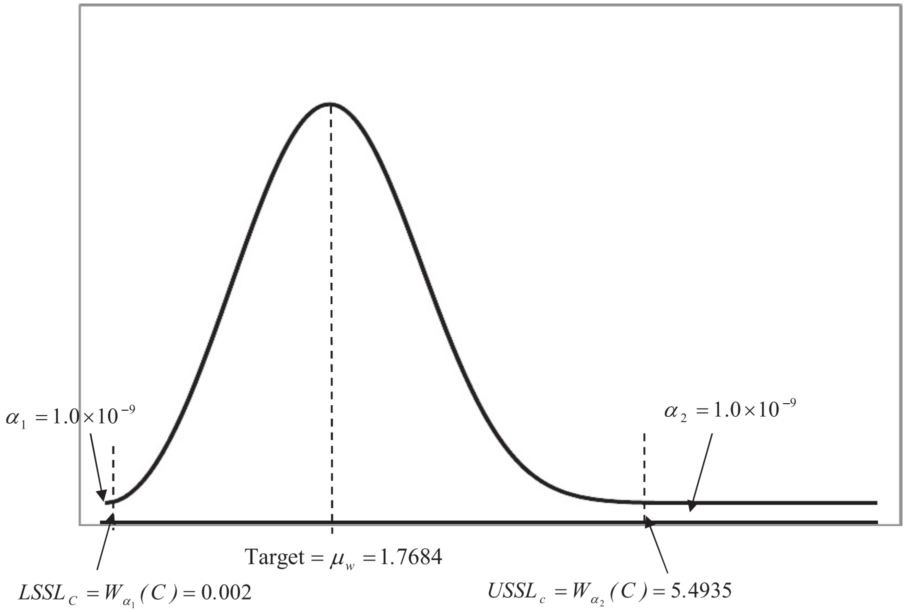

Figure 3. Proposed Six Sigma Weibull process with mean centered at target $\mu_w = 1.7684$ with two-sided Six Sigma limits $\mathrm{LSSL}_c = 0.002$ and $\mathrm{USSL}_c = 5.4935$ obtained by fixing $\alpha_1 = 1.00 \times 10^{-9}$ and $\alpha_2 = 1.00 \times 10^{-9}$ for a two-sided Weibull distribution with parameters $k = 3$ and $\beta = 2$

In order to better understand the results obtained in Table 4, we have presented Figure 3 that shows how the Six Sigma limits ($LSSL_c$ and $USSL_c$), the target and $\alpha_1 = \alpha_2 = 1.00\times10^{-9}$ can be graphically represented with SQL of 6 for a centered Weibull distribution with $k = 3$ and $\beta = 2$. Similar representation can be done for other SQLs also, but for want of space those graphs are not represented here.

As discussed in Section 3, for the shifted case of Weibull process, first we have computed the shifted mean $\bar{X}_w$ and then using $\bar{X}_w$ and σ_w, the parameters k' and β' of the shifted Weibull distribution are computed as follows:

$$k' = \left(\frac{\sigma_w}{\bar{X}_w}\right)^{-1.086} \quad \text{and} \quad \beta' = \frac{\bar{X}_w}{\Gamma(1 + 1/k')} \tag{24}$$

The values of k' and β' corresponding to various Weibull parameter settings are computed and given in Tables 5 and 6 for right and left shifts respectively. It may be noted that in Table 6, the parameter settings where the shifted mean is greater than zero only are computed. Now, for illustration purpose, the SQLs for the right-shifted Weibull process with parameters $k' = 4.815$ and $\beta' = 3.012$ are computed and shown in Table 7.

It is observed that there is a vast difference between the SQL values computed with the assumption that the underlying distribution is normal while the original distribution is Weibull. For example, in Table 4, for a centered Six Sigma process with equal tail probabilities, the Weibull distribution with parameters $k = 3$ and $\beta = 2$ shows the SQL of 2.767351 from target to LSSL and 5.835971 from the target to USSL with an average SQL of just 4.3017. Similarly, in Table 7, for a right-shifted Six Sigma process, the Weibull distribution with parameters $k' = 4.815$ and $\beta' = 3.012$ shows the SQL of 1.249 from target to LSSL and 4.214 from the target to USSL with an average SQL of just 2.73.

In order to better understand the results presented in Table 7, we have presented Figure 4 that shows how the Six Sigma specification limits ($LSSL_c$ and $USSL_c$), the target and shifted mean can be graphically represented for a sigma level of 6 for a right-shifted Weibull distribution. It may be noted that since mean has shifted from the target of $\mu_w = 1.7684$ to the

Table 5.
Values of k' and β' corresponding to various Weibull parameter settings (right shift)

Centered case		Right shifted case				
k	β	$\bar{X}_w$	σ_w^2	σ_w	k'	β'
0.5	2.0	17.416	80.000	8.944	2.062	19.66
1.0	2.0	5.000	4.000	2.000	2.705	5.622
1.5	2.0	3.643	1.501	1.225	3.266	4.063
2.0	2.0	3.161	0.857	0.926	3.794	3.498
2.5	2.0	2.913	0.576	0.759	4.329	3.198
3.0	2.0	2.759	0.421	0.649	4.815	3.012

Table 6.
Values of k' and β' corresponding to various Weibull parameter settings (left shift)

Centered case		Left shifted case				
k	β	$\bar{X}_w$	σ_w^2	σ_w	k'	β'
0.5	2.0	–	–	–	–	–
1.0	2.0	–	–	–	–	–
1.5	2.0	–	–	–	–	–
2.0	2.0	0.384	0.857	0.926	0.383	0.102
2.5	2.0	0.636	0.576	0.759	0.825	0.573
3.0	2.0	0.812	0.421	0.649	1.276	0.876

SQL of normal process	LSSL $W_{\alpha_1}(C)$	USSL $W_{\alpha_2}(C)$	α_1'	α_2'	LSQL K_{s1}	USQL K_{s2}	Average SQL of Weibull process
6	0	5.43156	0	3.37375×10^{-8}	1.252	4.118	2.68
	0.00159	5.45744	0.000312706	2.26436×10^{-8}	1.249	4.158	2.70
	0.002	5.49350	0.000419414	1.28343×10^{-8}	1.249	4.214	2.73
	0.00229	5.55408	0.000498766	4.78610×10^{-9}	1.248	4.307	2.78
	0.00252	5.68986	0.000563749	4.49531×10^{-10}	1.248	4.516	2.88
5	0	4.86259	0	4.13765×10^{-5}	1.252	3.241	2.25
	0.01047	4.89481	0.003484892	2.98524×10^{-5}	1.236	3.291	2.26
	0.01319	4.93952	0.004680695	1.87177×10^{-5}	1.231	3.360	2.30
	0.0151	5.01418	0.005562821	8.27435×10^{-5}	1.229	3.475	2.35
	0.01661	5.05205	0.006282330	5.37074×10^{-5}	1.226	3.534	2.38
4	0	4.26049	0	0.004809847	1.252	2.314	1.78
	0.05022	4.30234	0.025639502	0.003718018	1.174	2.378	1.78
	0.06328	4.35998	0.034314398	0.00256654	1.154	2.467	1.81
	0.07244	4.45513	0.04066347	0.001334274	1.140	2.614	1.88
	0.07973	4.51605	0.045850006	0.000852628	1.129	2.708	1.92
3	0	3.61691	0	0.088584141	1.252	1.322	1.29
	0.17546	3.67463	0.120862606	0.073094049	0.981	1.411	1.20
	0.22109	3.75302	0.159002398	0.055233893	0.911	1.532	1.22
	0.25311	3.87991	0.186083576	0.033386091	0.862	1.727	1.29
	0.27861	4.01854	0.207699939	0.017838142	0.822	1.941	1.38

Table 7. Computation of α_1' and α_2', and sigma quality levels for Weibull distribution with $k' = 4.815$ and $\beta' = 3.012$ for right shift

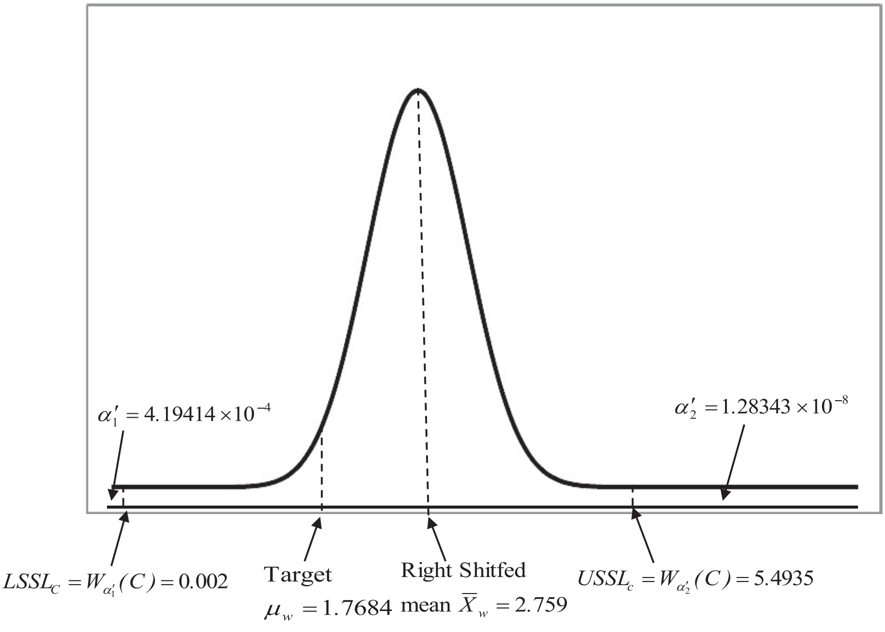

Figure 4. Proposed Six Sigma Weibull process with mean shifted to $\bar{X}_w = 2.759$ from the target $\mu_w = 1.7684$ with two-sided Six Sigma specification limits $LSSL_c = 0.002$ and $USSL_c = 5.4935$. The shifted parameters are $k' = 4.815$ and $\beta' = 3.012$

mean $\bar{X}_w = 2.759$, the parameters become $k' = 4.815$ and $\beta' = 3.012$ and hence the distribution changes and moves toward right. Therefore, the probability values α_1 and α_2 will become $\alpha_1' = 4.19414\times10^{-4}$ and $\alpha_2' = 1.28343\times10^{-8}$ respectively. In fact α_1' and α_2' are computed by setting

$$\alpha_1' = P\left(X < 0.002/k' = 4.815, \beta' = 3.012\right) \text{ and}$$

$$\alpha_2' = P\left(X > 5.4935/k' = 4.815, \beta' = 3.012\right)$$

Refer to Eqn (13). Similar representation can be done for other SQLs and also for left-shifted Weibull process, but for want of space those diagrams are not represented here.

It can be seen in Weibull process with both centered and shifted cases that the SQLs decrease in tune with the decrease of SQLs in normal case (refer to Tables 4 and 7) and hence DPMOs/EGPMOs also follow the pattern. Table 8 shows DPMO/EGPMO values based on Normal and Weibull distributions with centered and shifted cases. Similar computations can be obtained for a left-shifted Weibull process also and are not given for want of space.

4.4 Results based on simulation study

For each of the six data sets, 1,000 Weibull observations are simulated using the function $\beta\times(-\text{LN}(1-\text{RAND}))\wedge(1/\alpha)$ in Microsoft Office Excel 2010. This large set is considered to ensure large sample conditions, and hence for reasonably more accurate results. Now, the mean μ_w and variance σ_w^2 are estimated using (3) and (4) and are given in Table 9. This is justified by comparing the closeness of the mean and variance computed based on the parameter values (refer to Table 2) and those based on the simulated data (refer to Table 9).

SQL of normal process	Normal (DPMO/EGPMO) Centered	Normal (DPMO/EGPMO) Shifted	Weibull (DPMO/EGPMO) Centered Left tail	Weibull Centered Right tail	Weibull Right shifted Left tail	Weibull Right shifted Right tail
6	0.002	3.4	0	0.0020	0	0.03373752
	0.002	3.4	0.0005	0.0015	312.71	0.02264355
	0.002	3.4	0.0010	0.0010	419.41	0.01283430
	0.002	3.4	0.0015	0.0005	498.77	0.00478610
	0.002	3.4	0.0020	0	563.75	0.00044953
5	0.57	233	0	0.5733	0	41.38
	0.57	233	0.1433	0.4300	3484.89	29.85
	0.57	233	0.2867	0.2867	4680.69	18.72
	0.57	233	0.4300	0.1433	5562.82	8.27
	0.57	233	0.5733	0	6,282.33	5.37
4	63	6,200	0	63.34	0	4809.85
	63	6,200	15.84	47.51	25639.50	3718.02
	63	6,200	31.67	31.67	34314.40	2,566.54
	63	6,200	47.51	15.84	40663.47	1,334.27
	63	6,200	63.34	0	45850.01	852.63
3	2,700	66803	0	2,699.80	0	88584.14
	2,700	66803	674.95	2024.85	120862.61	73094.05
	2,700	66803	1349.90	1349.90	159002.40	55233.89
	2,700	66803	2024.85	674.95	186083.58	33386.09
	2,700	66803	2699.80	0	207699.94	17838.14

Table 8. DPMO/EGPMO based on normal and Weibull distributions with centered and right-shifted cases (refer to Tables 4 and 7 for SQLs of Weibull Process)

In this section, we present the results that are observed from a set of 1,000 observations simulated from Weibull distribution with parameters $k = 3$ and $\beta = 2$. The DPMO/EGPMO values are computed using Eqn (20) through (22) for both centered and shifted Weibull processes. The results are given in Table 10. It may be recalled that the values falling below LSSL or above USSL will show either DPMO or EGPMO values depending upon where the process specification is HTB or LTB type. Accordingly, in Table 10, we observed from the data generated that no values are falling below LSSL and above USSL of the Weibull distribution with parameters $k = 3$ and $\beta = 2$ corresponding to SQLs of 5 and 6 of normal distribution. However, there are few observations falling below LSSL and above USSL of the Weibull distribution corresponding to SQLs of 3 and 4 of normal distribution. Table 10 also shows the DPMO/EGPMO values as well. Now, for SQL of 3 in Table 10, last row under shifted process, if the quality characteristic is LTB type, then the EGPMO = 2000 and DPMO = 21000 where as if the quality characteristic is HTB type, then we have DPMO = 2000 and EGPMO = 21000.

Dataset	Parameters k	β	Mean and variance based on simulated data μ_w	σ_w^2	σ_w
1	0.5	2	3.97	88.85	9.43
2	1.0	2	1.99	3.76	1.94
3	1.5	2	1.80	1.41	1.19
4	2.0	2	1.77	0.89	0.94
5	2.5	2	1.77	0.54	0.76
6	3.0	2	1.77	0.41	0.64

Table 9. Mean and variance of Weibull distribution based on simulated data

	Centered process DPMO/EGPMO		Shifted process DPMO/EGPMO			
			Left shift		Right shift	
Sigma level	Left	Right	Left tail	Right tail	Left tail	Right tail
6	0	0	0	0	0	0
	0	0	0	0	0	0
	0	0	0	0	0	0
	0	0	1,000	0	0	0
	0	0	2000	0	0	0
5	0	0	0	0	0	0
	0	0	4000	0	0	0
	0	0	5000	0	0	0
	0	0	6,000	0	0	0
	0	0	7000	0	0	0
4	0	0	0	0	0	6,000
	0	0	27000	0	0	6,000
	0	0	41000	0	0	4000
	0	0	46000	0	0	3000
	0	0	54000	0	0	1,000
3	0	0	0	1,000	0	94000
	1,000	0	143000	1,000	1,000	74000
	1,000	*0*	*175000*	*1,000*	*2000*	*62000*
	1,000	0	205000	1,000	2000	37000
	1,000	0	228000	1,000	2000	21000

Table 10. Observed number of DPMO/EGPMO based on the simulated data (Weibull with $k = 3$ and $\beta = 2$)

In Table 10 for SQL of 3 case with equal tail probabilities (boldened row), if the quality characteristic is LTB type, then under right-shifted Weibull process, we have EGPMO = 2000 and DPMO = 62000, whereas if the quality characteristic is HTB type, then we have DPMO = 2000 and EGPMO = 62000. However, in case of normal process (refer to Table 8), for three sigma shifted case, we have EGPMO = 66803 and DPMO = 0 or DPMO = 66803 and EGPMO = 0 for LTB and HTB cases respectively. Though this trend is similar to those reported for exponential and gamma cases, there is a vast difference in the magnitude of DPMO/EGPMO (refer to Aldowaisan *et al.*, 2015 and Hassan *et al.*, 2018).

If we let $p_1 = 2000\times10^{-6}$ and $p_2 = 62000\times10^{-6}$, then using the inverse Weibull function IDF.WEIBULL(p, k, β)available in SPSS where p may be either $p_1$or p_2 given in Eqn. (22), the shifted LSSL and USSL, say $W_{\alpha_1'}(S)$ and $W_{\alpha_2'}(S)$ can be estimated as

$$W_{\alpha_1'}(S) = \text{IDF.WEIBULL}\left(2000\times10^{-6}, 4.815, 3.012\right) = 0.612$$

$$W_{\alpha_2'}(S) = \text{IDF.WEIBULL}\left(62000\times10^{-6}, 4.815, 3.012\right) = 6.762$$

Therefore, the corresponding actual SQLs $K_{sql(1)}$ and $K_{sql(2)}$ can be computed as

$$K_{sql1} = \frac{2.759 - 0.612}{0.649} = 3.308 \text{ and } K_{sql2} = \frac{6.762 - 2.759}{0.649} = 6.168$$

with an average SQL of $3.308 + 6.168/2 = 4.738$.

Therefore, if the process is assumed to be a three sigma process under normality assumption, the same will be 4.738 sigma process under Weibull distribution on the average.

Similarly, if the quality characteristic is LTB type, then under left shifted Weibull process, we have EGPMO = 175000 and DPMO = 1,000, whereas if the quality characteristic is HTB type, then we have DPMO = 175000 and EGPMO = 1,000. For normal process, there will not be any change in DPMO and EGPMO that were computed for right shift case due to symmetric property. The actual SQLs $K_{sql1}$and K_{sql2} for this left shift case can be computed as

$$K_{sql1} = \frac{0.814 - 0.612}{0.649} = 0.311 \text{ and } K_{sql2} = \frac{6.762 - 0.814}{0.649} = 9.164$$

with an average SQL of $(0.311 + 9.164)/2 = 4.738$. It is interesting to note that the average SQL is same for both right and left shift cases while the LSQL and USQL are different.

5. Discussion and conclusion

Introduced by Motorola, Six Sigma quality program has gained momentum as the companies could realize quality improvements and achieved business excellence. A typical Six Sigma quality program has a goal of 3.4 DPMO – the key metrics. While Six Sigma metrics have been studied by researchers in detail for normal distribution-based data, there are situations where the under lying distribution is non-normal. If the procedure under normality assumption is applied to non-normal case, it is argued that it may result in erroneous outcomes. Many authors have attempted to apply normal approximation to transform the non-normal data before determining the Six Sigma metrics. Particularly, such an approach makes the target of the non-normal process as zero which is again not practical in many situations where HTB and LTB type quality characteristics are in use. In the proposed method, we set actual target value rather than assuming zero as the target value. For example, the mean time (target) for clearing the bills cannot be set as zero though it is preferred. A minimum average time (target) is required.

It is suggested that in case of non-normal process, there is a scope for determining metrics (SQL, DPMO and EGPMO) taking into consideration the true distribution itself instead of normal approximation and this is the primary motivation for the authors to take up this research. In this paper, we have developed a theoretical procedure for studying the Six Sigma metrics for two-parameter Weibull distribution that is useful in many life test data analyses. We have considered the distribution as it is without making any normal approximation.

In the proposed approach, the probabilities under different normal process-based SQLs are matched to first determine the Six Sigma specification limits (LSSL and USSL) for a two-parameter Weibull distribution by setting different values for the shape parameter k and the scaling parameter β. Then, the USQL and LSQL values are obtained for the Weibull distribution with centered and shifted cases. For want of space, we presented the numerical results for Six Sigma metrics of centered Weibull distribution with parameter $k = 3$ and $\beta = 2$ and shifted Weibull distribution with parameters $k' = 4.815$ and $\beta' = 3.012$. The values for other sets of these parameters are also studied and can be obtained from the authors on request.

We also have simulated a set of 1,000 values from this Weibull distribution for both centered and shifted cases to evaluate DPMO/EGPMO values. In fact, due to the computational complexity involved, particularly for shifted cases, and without loss of generality, we retained standard deviation and then allowed mean to shift up to ± 1.5 times of standard deviation. As discussed in Section 4, the SQLs are slightly lesser than that of normal process when the data are assumed to follow two-parameter Weibull distribution. From the numerical evaluations, it is observed that the computation of SQLs for Weibull distribution is not straightforward. Unlike the normal process, in case of Weibull process the SQLs keep changing based on the nature of the parameters of the distribution and hence the DPMO and EGPMO values. Therefore one has to be cautious whenever a shift in the mean is noticed under the Weibull distribution.

We believe that the procedure developed here for determining Six Sigma metrics for Weibull distribution is new to the practitioners and is not only useful to the practitioners but will also serve to motivate the researchers to do more work in this field of research. This is mainly due to the fact that we have incorporated HTB and LTB cases with actual target value to determine DPMO and EGPMO which is more practical. The proposed procedure involves two-parameter Weibull distribution whose parameters change every time the mean changes (shifts in our case). We have addressed this issue clearly for the benefit of the researchers and practitioners of Six Sigma. The paper presented here has some limitations since it is based on certain parameter values though we have enough justification for the patterns in SQLs. A variety of parameter settings may further help to generalize the findings. Also, it is observed that in many situations, Weibull Generalized Exponential distribution (WGED) provides better fit than the Weibull distribution. As a future work, we would like to take into account the advantages of WGED and study the Six Sigma metrics for the same.

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