SOLITON RESONANCES IN HELIUM FILMS

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The phenomenon of two-soliton resonances of the Kadomtsev–Petviashvilli equation for the superfluid surface density fluctuation in He films is studied. The velocity of the resonant soliton is obtained.

Recently the study of soliton propagation in helium films has received much attention. The intrinsic nonlinearity in this system arising from the van der Waals potential due to the substrate suggests the existence of solitonic behaviour. Several experiments [1] in ^{4}He films indicate the propagation of undistorted waves at very low temperatures. These finite amplitude effects are not explained in terms of a linearized theory. It was shown by Huberman [2] that the dynamics of the superfluid density is governed by the Korteweg–de Vries (KdV) equation. He predicted the existence of gapless solitons made up of superfluid condensate. He also obtained the conditions for the propagation of a solitary wave in such films.

Starting from the hydrodynamics Rutledge et al. [3] and Biswas and Warke [4] were able to confirm theoretically the predictions of Huberman about the existence of solitons in superfluid films. The nonlinear modes of thin helium films have also been studied by Nakajima et al. [5,6] using two-fluid hydrodynamics and standard nonlinear techniques. They discussed the experimental conditions that should be appropriate for the search of solitons in both very thin and saturated He films, predicting what should be the signature of a KdV soliton in a realistic experiment. Condat and Guyer [7] discussed the possibility of propagating KdV solitons in superfluid ^{4}He overlayed by a ^{3}He film. They analyzed the various dispersive and nonlinear contributions to the equation for the various nonlinear modes.

The nonlinear time evolution of the condensate wavefunction in superfluid films was studied by Kurihara [8] for a model which incorporates van der Waals potential due to the substrate in its fully nonlinear form, and a surface tension term. By numerical methods it was shown that even under strong nonlinearity there exist quite stable "quasi-solitons", which are bound states of localized excitations of amplitude and phase of the condensate. Kurihara [9] confirmed that the "quasi-solitons" found in the numerical work are stable at least in the asymptotic situation where quasi-solitons are essentially nonoverlapping. It was shown explicitly that in the small amplitude regime, the soliton reduces to the KdV one-soliton solution.

Biswas and Warke [10] studied the possibility of having two-dimensional nonlinear excitations in a thin superfluid ^{4}He film. They obtained the Kadomtsev–Petviashvilli (KP) equation for the superfluid surface density fluctations and concluded that two-dimensional localized waves (lumps) that decay algebraically in all horizontal directions should be detectable at very low temperatures.

In this paper we investigate the phenomenon of resonance [11,12] of solitons in superfluid ^{4}He films. We start with the nonlinear equation for the superfluid surface density obtained by Biswas and Warke [10]. This can be written as a KP equation,

$$[U_{\bar{t}} + 6UU_{\bar{x}} + U_{\bar{x}\bar{x}\bar{x}}] - U_{\bar{y}\bar{y}} = 0 , \qquad (1)$$

where $U = (\rho_0 - 3a)\rho_1/12(a + \rho_0)$, ρ_1 is the fluctuation in the uniform surface density ρ_0, $\bar{x} = k_0(x + C_3 t)$, $\bar{y} = \sqrt{2}\,k_0 y$, $\bar{t} = C_3 k_0 t$, $k_0^2 = 8m^2C_3^2/(\hbar^4 +$

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$4mB\rho_0$), $C_3 = [3A\rho_0/m(a+\rho_0)^4]^{1/2}$ is the third sound velocity, a and A are the constants of the van der Waals interaction, m is the mass of the helium atom and B is the surface tension constant.

The KP equation in the form given by eq. (1) has been discussed in great detail by Satsuma and Ablowitz [13]. The one-soliton solution has the form

$$U = \tfrac{1}{2}k^2 \operatorname{sech}^2\eta, \tag{2}$$

where

$$\eta = \tfrac{1}{2}k[\bar{x} + p\bar{y} - (k^2 - p^2)\bar{t}] + \eta^{(0)},$$

and k and kp are the components of linear momentum along the x and y direction respectively.

The two-soliton solution is given by

$$U = 2(\log f_2)_{\bar{x}\bar{x}}, \tag{3}$$

where

$$f_2 = 1 + \exp 2\eta_1 + \exp 2\eta_2 + A_{12}\exp[2(\eta_1+\eta_2)],$$

$$\eta_i = \tfrac{1}{2}k_i[\bar{x} + p_i\bar{y} - (k_i^2 - p_i^2)\bar{t}] + \eta_i^{(0)},$$

$$A_{12} = \frac{3(k_1-k_2)^2 + (p_1-p_2)^2}{3(k_1+k_2)^2 + (p_1-p_2)^2}.$$

Soliton resonances occur [11] when $A_{12} = 0$ or ∞, i.e., for

$$3(k_1 \pm k_2) + (p_1 - p_2)^2 = 0. \tag{4}$$

The plus sign refers to the case $A_{12} = \infty$ and is called plus resonance, and the other case ($A_{12} = 0$) is called minus resonance. The resonance phenomenon can be best understood by the asymptotic behaviour of the two solitons under the above conditions.

$$\begin{aligned} U &= U^{(1)} + U^{(2)} = \tfrac{1}{2}k_1^2\operatorname{sech}^2\eta_1 + \tfrac{1}{2}k_2^2\operatorname{sech}^2\eta_2, & y \to -\infty, \\ &= U^{(1+2)} = \tfrac{1}{2}(k_1+k_2)^2\operatorname{sech}^2(\eta_1+\eta_2), & y \to +\infty, \end{aligned} \tag{5a}$$

$$\begin{aligned} U &= U^{(1+2)} = \tfrac{1}{2}(k_1+k_2)^2\operatorname{sech}^2(\eta_1+\eta_2), & y \to -\infty, \\ &= U^{(1)} + U^{(2)} = \tfrac{1}{2}k_1^2\operatorname{sech}^2\eta_1 + \tfrac{1}{2}k_2^2\operatorname{sech}^2\eta_2, & y \to +\infty. \end{aligned} \tag{5b}$$

The above two resonances correspond to plus resonance. The minus resonances are given by

$$\begin{aligned} U &= U^{(1)} = \tfrac{1}{2}k_1^2\operatorname{sech}^2\eta_1, & y \to -\infty, \\ &= U^{(2)} = \tfrac{1}{2}k_2^2\operatorname{sech}^2\eta_2, & y \to +\infty, \\ &= U^{(1-2)} = \tfrac{1}{2}(k_1-k_2)^2\operatorname{sech}^2(\eta_1-\eta_2), & x \to +\infty, \end{aligned} \tag{6a}$$

$$\begin{aligned} U &= U^{(2)} = \tfrac{1}{2}k_2^2\operatorname{sech}^2\eta_2, & y \to -\infty, \\ &= U^{(1)} = \tfrac{1}{2}k_1^2\operatorname{sech}^2\eta_1, & y \to +\infty, \\ &= U^{(1-2)} = \tfrac{1}{2}(k_1-k_2)^2\operatorname{sech}^2(\eta_1-\eta_2), & x \to -\infty. \end{aligned} \tag{6b}$$

So the resonant soliton, in general, can be written in the form

$$U^{(1\pm2)} = \tfrac{1}{2}(k_1 \pm k_2)^2\operatorname{sech}^2(\eta_1 \pm \eta_2). \tag{7}$$

The amplitude and velocity [14] of the resonant soliton is given by

$$A_r = [6\rho_0(a+\rho_0)/(\rho_0 - 3a)](k_1 \pm k_2)^2, \tag{8}$$

$$V_r = C_3\,\frac{[k_1(k_1^2 - p_1^2) \pm k_2(k_2^2 - p_2^2)] - (k_1 \pm k_2)}{\{(k_1 \pm k_2)^2 + 2(k_1p_1 \pm k_2p_2)^2\}^{1/2}}. \tag{9}$$

We have shown that the phenomenon of soliton resonances can be observed in superfluid He films. One can visualize eq. (5a) as the formation of a single soliton as the result of a collision of two solitons. Eq. (5b) can be considered as the splitting of a single soliton into two solitons. Minus resonance can also be viewed in a similar fashion. It should be possible to observe these resonant solitons by measuring the velocities in the asymptotic limits.

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