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# Synchronization and control of chaos in coupled chaotic multimode Nd:YAG lasers

M.R. Parvathi [a,*], Bindu M. Krishna [b,1], S. Rajesh [a], M.P. John [a], V.M. Nandakumaran [a,2]

[a] International School of Photonics, Cochin University of Science and Technology, Cochin, India
[b] Sophisticated Test and Instrumentation Centre, Cochin University of Science and Technology, Cochin, India

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ABSTRACT

In this Letter we numerically investigate the dynamics of a system of two coupled chaotic multimode Nd:YAG lasers with two mode and three mode outputs. Unidirectional and bidirectional coupling schemes are adopted; intensity time series plots, phase space plots and synchronization plots are used for studying the dynamics. Quality of synchronization is measured using correlation index plots. It is found that for laser with two mode output bidirectional direct coupling scheme is found to be effective in achieving complete synchronization, control of chaos and amplification in output intensity. For laser with three mode output, bidirectional difference coupling scheme gives much better chaotic synchronization as compared to unidirectional difference coupling but at the cost of higher coupling strength. We also conclude that the coupling scheme and system properties play an important role in determining the type of synchronization exhibited by the system.



## 1. Introduction

Study of coupled chaotic systems, especially their synchronization properties has been an area of intense research for the last few decades. This phenomenon has found many applications in laser dynamics [1–4], electronic circuits [5,6], chemical and biological systems [7,8] and secure communication [9,10]. Dynamics of an array of coupled systems has also been studied for coupled maps [11–13], coupled semi conductor lasers [14], neural networks [15] and in the Josephson junction [16]. Many studies of coupled lasers have been motivated by the requirement of high power coherent sources. Intracavity doubled continuous wave infrared lasers have been identified as efficient source of coherent visible light. The development of high power laser diode arrays which provide a long lived, highly efficient pump source for the Nd:YAG laser, and the availability of doubling crystals with large nonlinear coefficients have permitted the development of cw visible sources with electrical efficiencies approaching 1%. These systems are one of the most efficient sources of cw coherent light in this portion of the visible spectrum [17]

Nd:YAG laser with intracavity KTP crystal is a system of special interest as the crystal can introduce periodic as well as chaotic fluctuations in the laser output. It was Baer who developed a deterministic rate equation model to explain the fluctuations [17]. It has been shown that the laser output can be stabilized through a sequence of reverse period doubling bifurcation by varying the orientation of KTP crystal and the laser cavity [18].

Several investigations have been carried out to understand the dynamics of coupled Nd:YAG lasers. Roy and Thornburg studied the coupling of two chaotic Nd:YAG lasers experimentally [4]. The coupling of lasers was achieved by generating two laser beams on the same crystal so that the intracavity laser fields overlap. They have also studied the chaotic behavior of two coupled single mode Nd:YAG lasers [19]. Coupled dynamics of two chaotic Nd:YAG lasers operating in three longitudinal modes were studied to show that the system goes from chaotic state to periodic and then to steady state as the coupling strength is increased [2].

Synchronization of chaotic systems has been widely studied because of its potential applications in the area of secure communication. Chaos based communication using Lorentz oscillators, Rossler systems and electronic circuits are possible at bandwidths of several tens of kilohertz [20–30]. By using the synchronization of optical chaos the available bandwidth can be extended up to

* Corresponding author. Tel.: +91 484 2575848; fax: +91 484 2576714.
*E-mail address:* parumeenu82@rediffmail.com (M.R. Parvathi).
1 Nee: V. Bindu.
2 Present address: Indian Institute of Science Education and Research, Thiruvananthapuram, India.

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hundreds of megahertz. Information encoding and decoding was studied in semiconductor lasers [31,32], solid state lasers [33], fiber-ring lasers [34] and in microchip lasers [35]. Quality of message encoding and decoding also depends on the choice of coupling scheme between the transmitter and the receiver laser systems. Proper selection of coupling scheme can enhance the privacy and security of communication.

Only single mode laser systems have been considered for single channel communication for the past several years. Recent studies reveal that multimode lasers can be used effectively for multichannel communication. Chaotic Nd:YAG lasers are ideal candidates for this purpose because of their inherent nonlinearity and multimode operation. Chaotic synchronization of multimode Nd:YAG lasers can be used in digital communication of two-dimensional messages [36] and in encrypted audio transmission [37]. In a multimode laser chaotic synchronization between corresponding cavity modes will be different from that between different cavity modes. Thus each pair of corresponding cavity modes can be used as a channel in optical communication [38]. To the best of our knowledge the effect of various coupling schemes on the dynamics of multimode Nd:YAG lasers and their synchronization properties have not yet been discussed in detail. In this Letter we present the results of our numerical studies on the effect of external electronic coupling scheme on the dynamics of two mode and three mode lasers. Direct and difference couplings are investigated separately for unidirectional and bidirectional schemes.

## 2. Laser model

For numerical work we consider an Nd:YAG laser with intracavity KTP crystal. Nd:YAG laser usually lases at 1064 nm in the infrared. The laser output is stable without the intracavity doubling crystal. However, when a nonlinear KTP crystal is inserted into the laser cavity, some of the infrared fundamentals are converted into green light (532 nm) by the process of second harmonic generation and sum frequency generation. Different longitudinal modes get coupled due to this sum frequency generation, which again produces deterministic intensity fluctuations in the laser output. Therefore, with the crystal inserted into the laser cavity, the output intensity exhibits periodic and chaotic fluctuations [39]. The system can be modeled by the rate equations for the intensity $I_k$ and gain $G_k$ for the $k$th longitudinal mode.

$$\tau_c \frac{dI_k}{dt} = \left(G_k - \alpha - g\varepsilon I_k - 2\varepsilon \sum_{j\neq k} \mu_{jk} I_j\right) I_k, \quad (1)$$

$$\tau_f \frac{dG_k}{dt} = \gamma - \left(1 + I_k + \beta \sum_{j\neq k} I_j\right) G_k, \quad (2)$$

$k = 1, 2, 3, \ldots$ are the mode numbers.

In Eqs. (1) and (2) $\tau_c$ is the cavity round trip time, $\tau_f$ is the fluorescence life time, $\alpha$ is the cavity loss parameter, $\beta$ is the cross saturation parameter, $\varepsilon$ is the nonlinear gain coefficient and $\gamma$ is the small signal gain related to pump rate. $g$ is a geometrical factor whose value depends on the angle between the YAG and KTP fast axes as well as on the phase delays due to their birefringence. The parameter values used in the numerical simulation are given in Table 1.

Two such lasers are coupled via external electronic coupling [2] whereby the pumping of each laser is modulated according to the output intensity of the other. The rate equations for the coupled system are given by

$$\tau_c \frac{dI_{k1}}{dt} = \left(G_{k1} - \alpha - g\varepsilon I_{k1} - 2\varepsilon \sum_{j1\neq k1} \mu_{j1k1} I_{j1}\right) I_{k1},$$

**Table 1**
Parameter values used in the numerical simulation of two coupled chaotic Nd:YAG lasers.

| | |
|---|---|
| $\tau_c$, cavity round trip time | 0.2 ns |
| $\tau_f$, fluorescence life time | 240 µs |
| $\alpha$, cavity loss parameter | 0.01 |
| $\beta$, cross saturation parameter | 0.7 |
| $\varepsilon$, non-linear gain coefficient | $5 \times 10^{-6}$ |
| $\gamma$, small signal gain related to pump rate | 0.05 |

$$\tau_f \frac{dG_{k1}}{dt} = \gamma_1 - \left(1 + I_{k1} + \beta \sum_{j1\neq k1} I_{j1}\right) G_{k1}, \quad (3)$$

$$\tau_c \frac{dI_{k2}}{dt} = \left(G_{k2} - \alpha - g\varepsilon I_{k2} - 2\varepsilon \sum_{j2\neq k2} \mu_{j2k2} I_{j2}\right) I_{k2},$$

$$\tau_f \frac{dG_{k2}}{dt} = \gamma_2 - \left(1 + I_{k2} + \beta \sum_{j2\neq k2} I_{j2}\right) G_{k2}. \quad (4)$$

The coupling is done by modifying the parameter $\gamma$. We have considered two types of couplings, namely direct coupling and difference coupling schemes. In the direct coupling scheme, the parameter $\gamma$ of laser 2 is modulated according to the total intensity of laser 1 and vice versa.

$$\gamma_1 = \gamma_b + K_1 \sum_{k2} I_{k2},$$

$$\gamma_2 = \gamma_b + K_2 \sum_{k1} I_{k1}. \quad (5)$$

On the other hand, in the difference coupling scheme the parameter is modulated according to the difference in intensities of the two lasers 1 and 2.

$$\gamma_1 = \gamma_b + K_1 \sum (I_{k2} - I_{k1}),$$

$$\gamma_2 = \gamma_b + K_2 \sum (I_{k1} - I_{k2}). \quad (6)$$

In Eqs. (5) and (6) $K_1$ and $K_2$ are the coupling constants and $\sum I_{k1}$ and $\sum I_{k2}$ are the total output intensities in all the modes for laser 1 and laser 2, respectively.

## 3. Numerical results and discussion

### 3.1. Dynamics of the system under unidirectional coupling

The coupling constants are set as $K_1 = 0$, $K_2 = K$ so that laser 1 is coupled to laser 2 but there is no backward coupling. We give the results of both direct and difference couplings for lasers with two mode and three mode output.

#### 3.1.1. Two mode case with unidirectional coupling

In this section we present the dynamics of two coupled Nd:YAG lasers each operating in two orthogonal longitudinal modes with unidirectional coupling between them. First we investigate the dynamics of the system under direct coupling scheme. Fig. 1 shows the time series for the total intensities for the lasers 1 and 2 for the coupling strength $K = 0.002$. It is seen that there is amplification in the output intensity of laser 2 as the coupling strength is increased. Fig. 2 shows the time series for the total intensities for the lasers 1 and 2 for $K = 0.8$.

At higher coupling strength ($K = 1$) the two lasers achieve phase synchronization. The phase of the chaotic time series is calculated using the method of Hilbert Transform [40]. In order to show the phase synchronization we calculate the similarity function defined by [41]

$$S(\tau) = \sqrt{\frac{\langle [x_2(t+\tau) - x_1(t)]^2 \rangle}{[\langle x_1^2(t)\rangle \langle x_2^2(t)\rangle]^{1/2}}}, \quad (7)$$

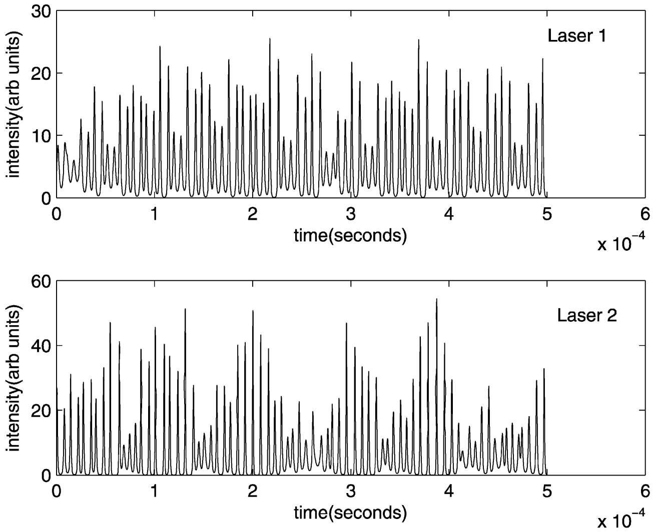


**Fig. 1.** Time series plots for laser 1 and laser 2 for the coupling strength $K = 0.002$ under unidirectional direct coupling scheme.

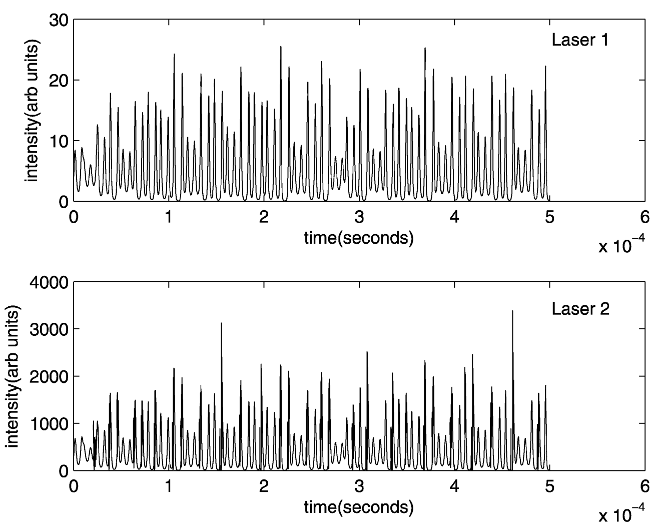


**Fig. 2.** Time series plots for laser 1 and laser 2 for the coupling strength $K = 0.8$ under unidirectional direct coupling scheme.

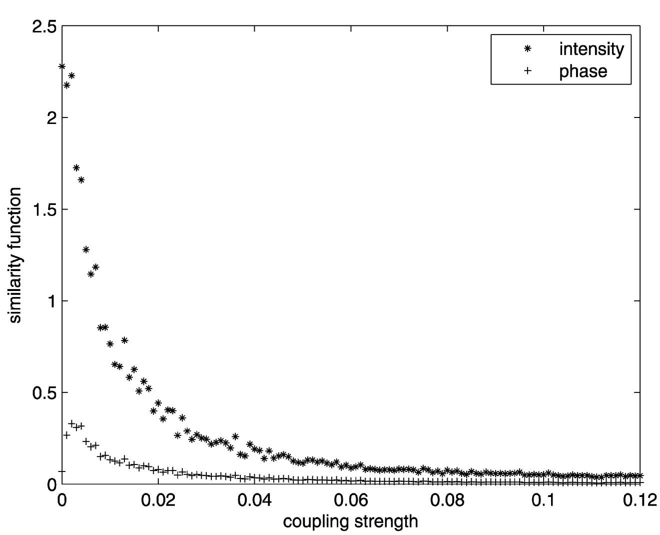


**Fig. 3.** Similarity function vs. coupling strength for both phase and intensity for two, coupled two mode chaotic lasers under unidirectional direct coupling scheme.

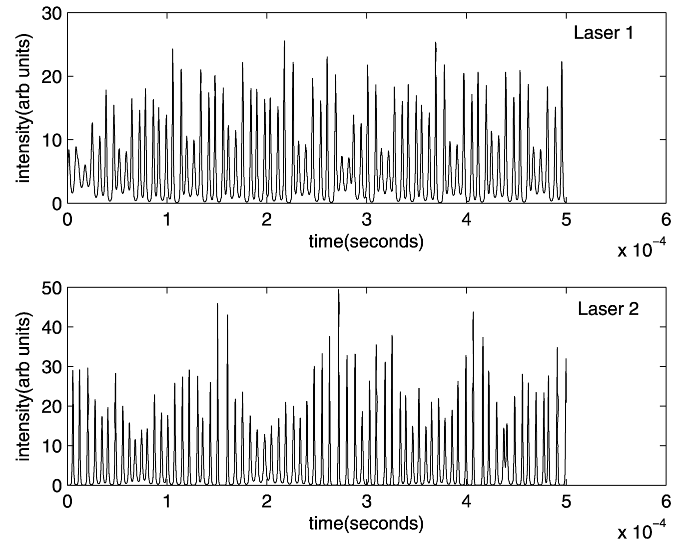


**Fig. 4.** Time series plots for laser 1 and laser 2 for the coupling strength $K = 0.002$ under unidirectional difference coupling scheme.

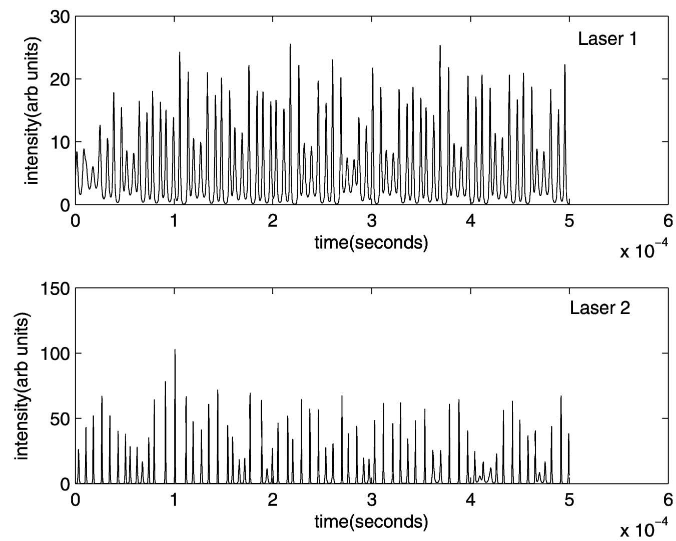


**Fig. 5.** Time series plots for laser 1 and laser 2 for the coupling strength $K = 0.006$ under unidirectional difference coupling scheme.

where $x(t)$ could either be the phase or intensity of the laser output.

For values of the coupling increasing from zero in small steps the similarity function for both phase and intensity are determined. Fig. 3 shows the variation of similarity function for intensity and phase with respect to increase in coupling strength. It is seen from the figure that for a large range of values of the coupling strengths the similarity function for the phases show very low values indicating the existence of phase synchronization in the system. The synchronization of intensities is not so pronounced for lower coupling strengths.

In the case of difference coupling there is no synchronization. However, amplification of the signal is present in this case also. Fig. 4 shows the intensity time series plots of laser 1 and 2 for the coupling strength $K = 0.002$. Amplification present is clear from the time series plots in Fig. 5 for the coupling strength $K = 0.006$.

*3.1.2. Three mode case with unidirectional coupling*

Here we consider two coupled Nd:YAG lasers each operating in three longitudinal modes, two of which are polarized parallel to

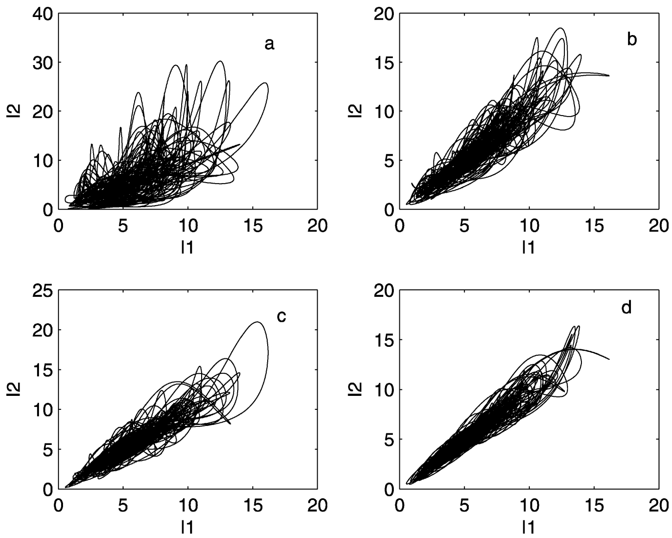


**Fig. 6.** Synchronization plots of two, coupled three mode chaotic lasers for various coupling strengths under unidirectional difference coupling scheme. (a) For coupling strength 0.02. (b) For coupling strength 0.04. (c) For coupling strength 0.05. (d) For coupling strength 0.055.

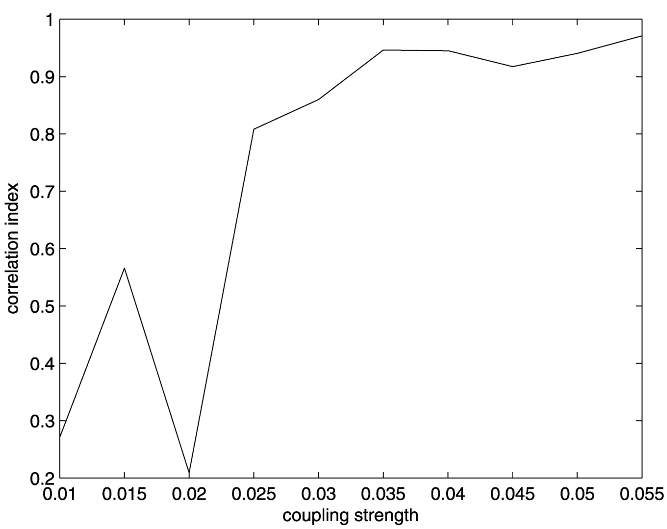


**Fig. 7.** Correlation index vs. coupling strength under unidirectional difference coupling scheme.

each other while the third mode is polarized in the orthogonal direction. When direct coupling is employed, the system shows amplification of the output intensity of the second laser as in the two mode case, but does not show any tendency to get synchronized. However, when the difference coupling scheme is used, the system shows synchronization. Fig. 6 show the synchronization plots for various coupling strengths.

The quality of synchronization is determined by the correlation index defined by [42]

$$\rho = \frac{\langle [x(t) - \langle x(t)\rangle][y(t) - \langle y(t)\rangle]\rangle}{\sqrt{\langle |x(t) - \langle x(t)\rangle|^2\rangle}\sqrt{\langle |y(t) - \langle y(t)\rangle|^2\rangle}} \tag{8}$$

where $x(t)$ could either be the phase or intensity of the laser output.

In Fig. 7 we plot correlation index, as a function of coupling strength. The maximum value of $\rho$ obtained is 0.9710 corresponding to a coupling strength 0.055.

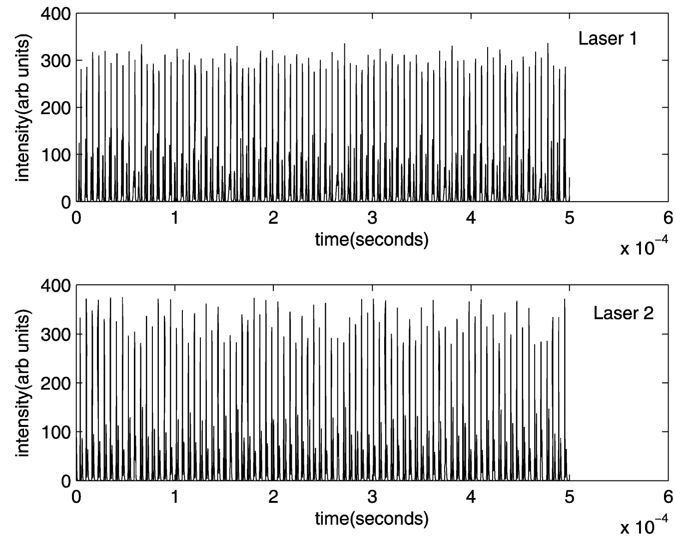


**Fig. 8.** Time series plots for lasers 1 and 2 for the coupling strength $K = 0.0072$ under bidirectional direct coupling scheme.

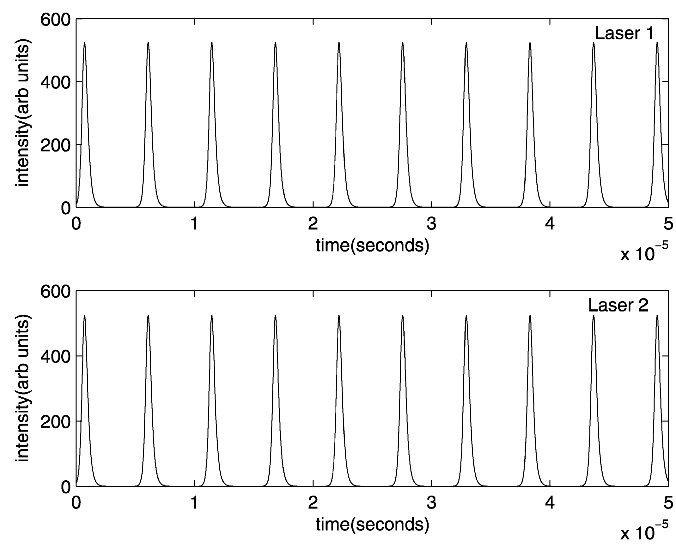


**Fig. 9.** Time series plots for lasers 1 and 2 for the coupling strength $K = 0.008$ under bidirectional direct coupling scheme.

### *3.2. Dynamics of the system under bidirectional coupling*

The coupling constants are set as $K_1 = K_2 = K$ so that laser 1 is coupled to laser 2 and vice versa. Here also we give the results of both direct and difference coupling schemes for lasers with two mode and three mode output.

#### *3.2.1. Two mode case with bidirectional coupling*

Dynamics of two coupled Nd:YAG lasers each operating in two orthogonal longitudinal modes with bidirectional coupling between them is studied. Under direct coupling scheme the two lasers remain chaotic up to $K = 0.0072$ after which they begin to be periodic. The two lasers exhibit period 1 oscillations in their output at $K = 0.008$. Even though the lasers exhibit periodic behavior above 0.0072, they are not synchronized. The two lasers get synchronized only at $K = 0.008$. We can also see an amplification in output intensity of both the lasers at $K = 0.008$. After $K = 0.008$, even though synchronization is lost for some values of coupling strength, for higher values of coupling strength they again get synchronized. Figs. 8 and 9 gives the time series plots for laser 1 and laser 2 for $K = 0.0072$ and $K = 0.008$ respectively.

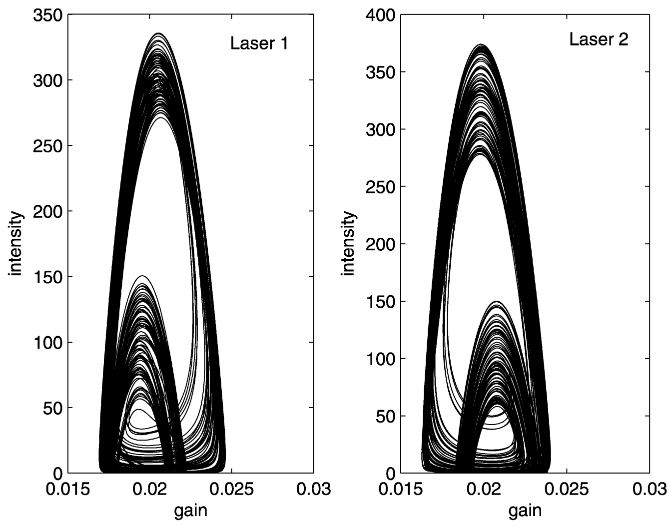


**Fig. 10.** Phase space plots for lasers 1 and 2 for the coupling strength $K = 0.0072$ under bidirectional direct coupling scheme.

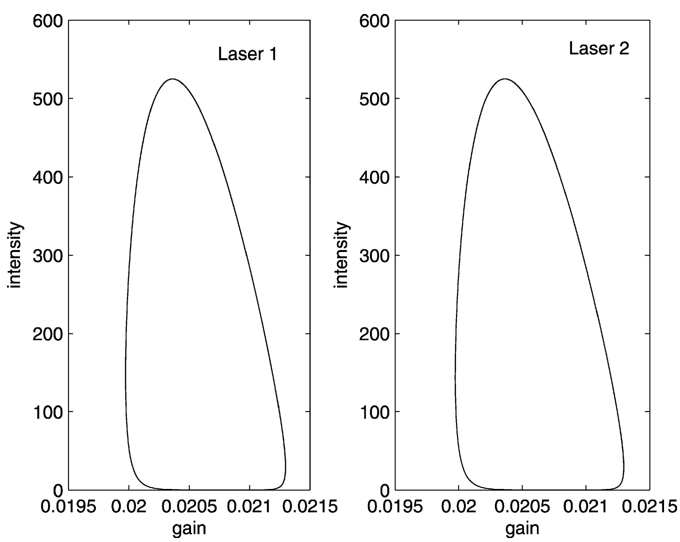


**Fig. 11.** Phase space plots for lasers 1 and 2 for the coupling strength $K = 0.008$ under bidirectional direct coupling scheme.

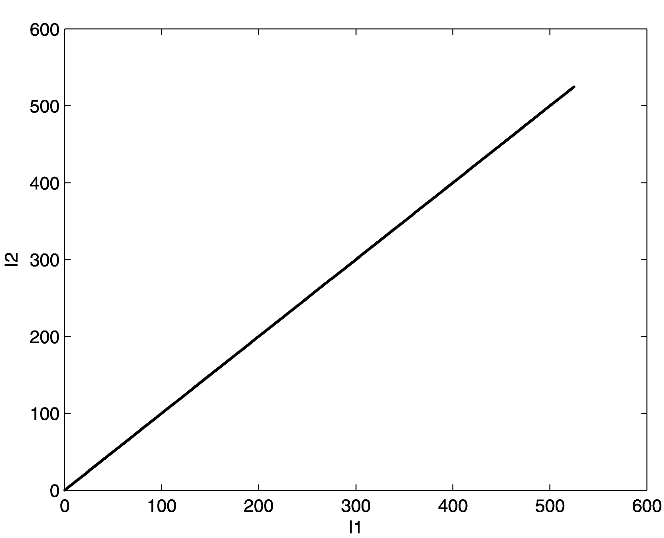


**Fig. 12.** Synchronization plot for lasers 1 and 2 for the coupling strength $K = 0.008$ under bidirectional direct coupling scheme.

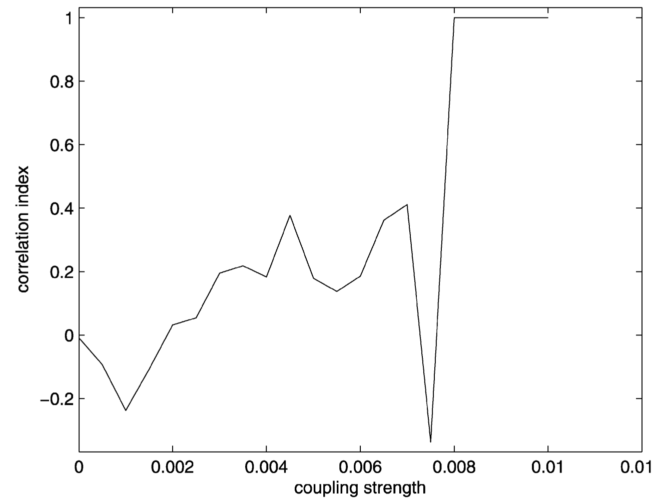


**Fig. 13.** Correlation index vs. coupling strength under bidirectional direct coupling scheme.

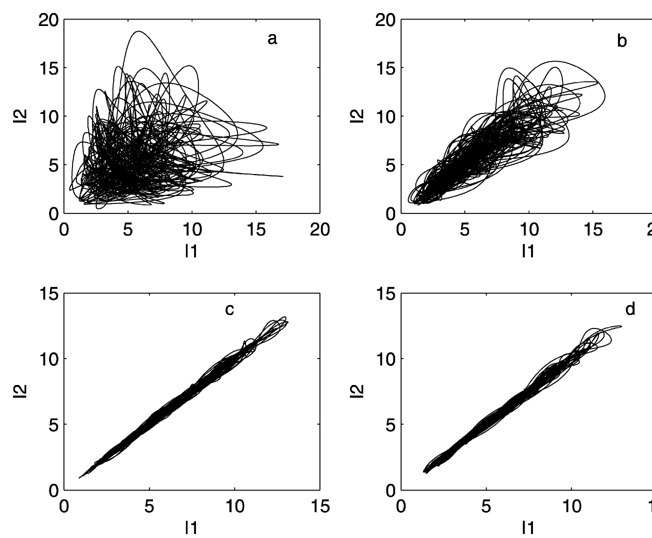


**Fig. 14.** Synchronization plots of two, coupled three mode chaotic lasers for various coupling strengths under bidirectional difference coupling scheme. (a) For coupling strength 0.004. (b) For coupling strength 0.02. (c) For coupling strength 0.085. (d) For coupling strength 0.101.

The corresponding phase space plots are given in Figs. 10 and 11. The synchronization plot for $K = 0.008$ is shown in Fig. 12.

In Fig. 13 we plot the correlation index $\rho$ as a function of coupling strength. $\rho$ is close to 1 for coupling strengths between 0.008 and 0.01.

However when we use difference coupling the outputs are neither stabilized nor synchronized.

### 3.2.2. Three mode case with bidirectional coupling

Here we consider two Nd:YAG lasers each operating in three longitudinal modes, two of which are polarized parallel to each other while the third mode is polarized in the orthogonal direction. It has been proved earlier that the lasers can be stabilized through direct coupling scheme and thus control chaos [2]. When the difference coupling scheme is used, it is seen that both the lasers remain chaotic throughout the entire range of coupling strengths. Though we cannot control chaos using this scheme it is found to be effective in synchronizing the lasers. The quality of synchronization is better as compared to unidirectional scheme. The synchronization plots for various coupling strengths are shown in Fig. 14.

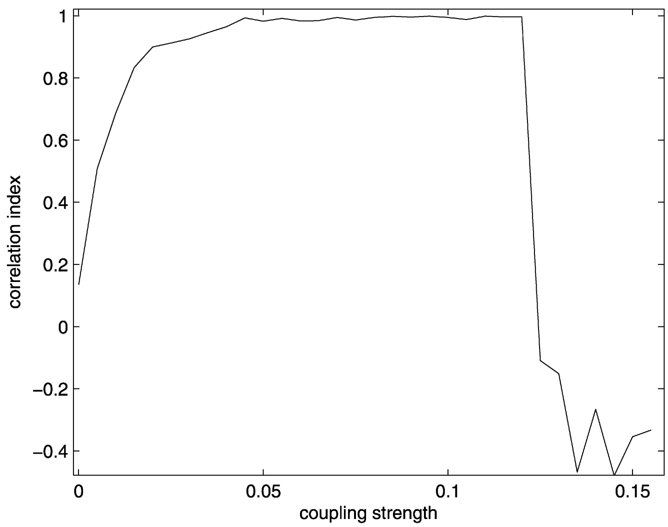


**Fig. 15.** Correlation index vs. coupling strength under bidirectional difference coupling scheme.

Here also the correlation index is calculated for various values of coupling strength and is plotted in Fig. 15. Correlation index increases with the coupling strength initially, reaches a maximum at the coupling strength 0.101 and decreases steeply for higher values of couplings. Maximum value of correlation index obtained is 0.9987 corresponding to the coupling strength 0.101. The sharp decrease in correlation index for higher values of couplings indicates the loss of synchronization in that region.

## 4. Conclusions

Dynamics of a system of two coupled chaotic multimode Nd:YAG lasers having two mode and three mode outputs was studied numerically. The lasers were coupled using external electronic coupling in which the pumping of each laser is modulated according to the output intensity of the other. Both bidirectional and unidirectional coupling strategies are employed. Intensity time series plots, phase space plots and synchronization plots are used to study the individual dynamics and synchronization properties.

We have found that for a system of coupled Nd:YAG lasers operating in two modes, complete synchronization between the lasers can be achieved using bidirectional direct coupling only. With this coupling scheme, we can control chaos and get amplification in output intensity of both the lasers. With bidirectional difference coupling, the lasers remain chaotic for the entire region of coupling strength. We cannot observe any amplification or synchronization under this scheme. With unidirectional direct coupling, phase synchronization between the lasers can be observed at higher coupling strengths. Amplification in output intensity of second laser is also observed in this region. Unidirectional difference coupling can give the same results except for any type of synchronization.

For systems operating in three mode, both unidirectional and bidirectional difference coupling schemes are effective in inducing complete chaotic synchronization between the lasers. Much better synchronization is obtained with bidirectional difference coupling, but at the cost of high coupling strength. With bidirectional direct coupling we can control chaos [2] while unidirectional direct coupling is not efficient in controlling chaos or synchronizing the lasers.

To conclude, our results corroborate the findings that the coupling strategies and system properties play an important role in determining the dynamics and synchronization phenomena exhibited [43]. Thus a deep knowledge about the system helps us in choosing the appropriate coupling scheme that gives the desirable output.

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